APEC 8213: Econometric Analysis III
Spring 2026
Professor: Paul Glewwe
Teaching Assistant: Matt Braaksma

Problem Set 1

Handed out: January 26, 2026
Due: February 4, 2026

1. Maximum Likelihood

(40 points total)

Let X be distributed as N(μ, σ^2). The unknown parameters are μ and σ^2.

a) Find the log-likelihood function $\ell_n(\mu, \sigma^2)$. **(8 points)**

b) Take the first order condition with respect to μ, and show that the solution for $\hat{\mu}$ does not depend on σ^2. **(10 points)**

c) Define the concentrated log-likelihood function $\ell_n(\hat{\mu}, \sigma^2)$. The concentrated log-likelihood substitutes the MLE $\hat{\mu} = \bar{x}$ from part (b) into the original log-likelihood, treating it as a function of σ^2 only. Take the first order condition with respect to σ^2, and find the MLE $\hat{\sigma}^2$ for σ^2. **(12 points)**

d) Generate 1000 observations from N$(\mu = 10, \sigma^2 = 4)$. **(10 points)**

i. Write an R function `loglik_normal(theta, y)` that computes the log-likelihood, where `theta = c(mu, sigma2)`.

ii. Use `optim()` to estimate μ and σ². Use starting values `theta_init = c(0, 1)` and `set.seed(8213)`.

iii. Compare your MLE estimates to:

- The true parameters (μ = 10, σ² = 4)
- The sample mean $\bar{y}$ and sample variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

Present your results in a table.

2. Binary Choice Logistic Distribution

(20 points total)

For the logistic distribution $\Lambda(x) = (1 + \exp(-x))^{-1}$, verify that

a) $\frac{d}{dx}\Lambda(x) = \Lambda(x)(1 - \Lambda(x))$. **(5 points)**

b) $h_{\text{logit}}(x) = \frac{d}{dx}\log\Lambda(x) = 1 - \Lambda(x)$. **(5 points)**

c) $H_{\text{logit}}(x) = -\frac{d^2}{dx^2}\log\Lambda(x) = \Lambda(x)(1 - \Lambda(x))$. **(5 points)**

d) $|H_{\text{logit}}(x)| \leq 1$. **(5 points)**

3. Binary Choice Estimation

(40 points total)

The data for Question 3 come from Grogger, J. (1991), "Certainty vs. Severity of Punishment," *Economic Inquiry* 29, 297-309. The dataset `grogger.dta` is available on Canvas.

Dataset description:

- Unit of observation: Individual men
- Sample: Men arrested at least once prior to 1986
- Time period: 1986 arrests and prior criminal history
- Key variables:
  - `narr86`: Number of arrests in 1986
  - `pcnv`: Proportion of prior arrests resulting in conviction (probability of conviction)

- `avgsen`: Average sentence length (months) from prior convictions
- `tottime`: Total time spent in prison since age 18 (months)
- `ptime86`: Months spent in prison during 1986
- `inc86`: Legal income in 1986 (hundreds of dollars)
- `black`: =1 if black
- `hispan`: =1 if Hispanic
- `born60`: =1 if born in 1960 or later

a) Define a binary variable, $arr86$, equal to unity if a man was arrested at least once during 1986, and zero otherwise. Estimate an LPM relating $arr86$ to $pcnv$, $avgsen$, $tottime$, $ptime86$, $inc86$, $black$, $hispan$, and $born60$. Report the usual and heteroskedasticity-robust standard errors. What is the estimated effect on the probability of arrest if $pcnv$ goes from .25 to .75? **(8 points)**

b) Test the joint significance of $avgsen$ and $tottime$, using a nonrobust and robust test. **(6 points)**

c) Now estimate the model by probit. At the average values of $avgsen$, $tottime$, $inc86$, and $ptime86$ in the sample, and with $black = 1$, $hispan = 0$, and $born60 = 1$, what is the estimated effect on the probability of arrest if $pcnv$ goes from .25 to .75? Compare this result with the answer from part a. **(8 points)**

d) For the probit model estimated in part c, obtain the percent correctly predicted. What is the percent correctly predicted when $narr86 = 0$? When $narr86 = 1$? What do you make of these findings? **(8 points)**

e) In the probit model, add the terms $pcnv^2$, $ptime86^2$, and $inc86^2$ to the model. Are these individually or jointly significant? Describe the estimated relationship between the probability of arrest and pcnv. In particular, at what point does the probability of conviction have a negative effect on probability of arrest? **(10 points)**